Object-Oriented Modelling of Hybrid Technical Systems

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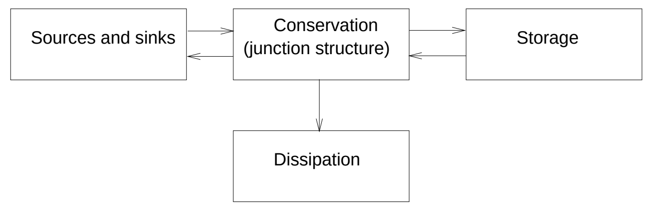
Lecture 5 – Bond Graph Modelling

- Introduction to Bond Graphs
- Bond Graph Elements
- Causality
- Modelling Example
- Analysis of Models
- Advanced Bond Graphs

Introduction to Bond Graphs

Invented by Henry Paynter at MIT, 1960.
Motivation:

- Energy based modelling (Lagrange)
- Power continuous, fulfills first law
- Analogies, mechanics - electrics
- Multidisciplinary, unified approach for modelling of mixed systems
- Non-causal, imposed by boundaries
- Graphical method, bonds describing energy interchange between systems

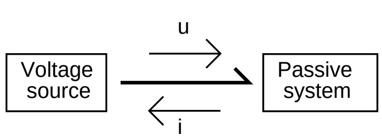


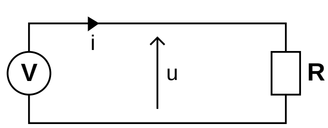
Introduction to Bond Graphs
Power Conjugate Variables

Energy interchange is a two way information flow. It is described by one effort and one flow variable.
Power conjugate, their product is power.

Example: An electrical connection is described by the voltage between the terminals and the current through them. The energy flowing between the two systems is

$$E = \int u \cdot i \, dt$$





Introduction to Bond Graphs
Power Conjugate Variables

In each engineering domain there is one pair of power conjugate variables.
Some common examples are:

Domain	e, effort	f, flow
Electric	u, voltage	i, current
Mechanics	F, force	v, velocity
Rotation	M, torque	ω, angular vel.
Hydraulic	p, pressure	q, volume flow
Thermal	T, temperature	$\dot{S}$, entropy flow

Bond Graph Elements
Basic Elements

- Sources/Sinks **Se** ⟶
  Active elements, that supplies/removes effort or flow.

- Resistance ⟶ **R**
  Passive dissipative elements. Removes energy and relates effort to flow.
  Constitutive relation:

$$e(t) = h(f(t))$$
$$f(t) = h^{-1}(e(t))$$
$$u(t) = R\,i(t)$$

Bond Graph Elements
Energy Storing Elements

In dynamical systems energy can be stored, there are two kinds of energy storage:

- Capacitor/Compliance (flow storage)
  ⟶ **C**

$$e(t) = G(\int f(t)dt)$$
$$u(t) = \frac{1}{C}\int i(t)dt$$
$$i(t) = C\,du/dt$$

- Inductance/Inertia (effort storage)
  ⟶ **I**

$$f(t) = G(\int e(t)dt)$$
$$v(t) = \frac{1}{m}\int F(t)dt$$
$$F(t) = m\,dv/dt$$

Bond Graph Elements
Junction elements

Elements to build model junction structure.
Requirements are:
power continuous, symmetrical and ability to be used as building blocks.

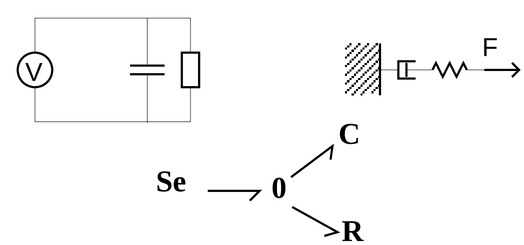
There are two unique three-port junctions:

- 0-junction Common effort, the flows sum to zero. Like a parallel connection in an electrical network.
- 1-junction Common flow, the efforts sum to zero. Like a closed series connection in an electrical network.

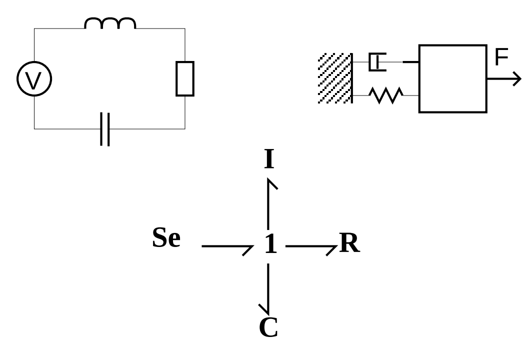
Alternative notation: p- and s-junctions
Mnemonic: 0 - p - e lower than 1 - s - f

Bond Graph Elements
Junction examples

Common effort:



Common flow:



Bond Graph Elements
Energy Transducers

Relaxing the symmetry requirement, two unique two-port junctions can be derived describing energy flowing from one physical domain into another.

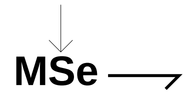
- A Transformer transforms effort in one domain into effort in another, e. g., force $\sim$ pressure in a hydraulic piston.
  $F = p\,A$
  

  $q = v\,A$
- A Gyrator relates effort in one domain with flow in another, e. g., current $\sim$ torque in an electrical motor.
  $M = k_m\,i$
  

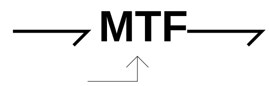
  $u = k_m\,\omega$

Bond Graph Elements
Modulated Elements

Some elements can be modulated by an external variable or signal.

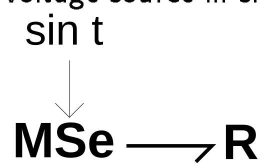


- Sources
  modulated by a signal generator or a control signal



- Transducers
  nonlinear transformations, modulated by for example angle or position

Example: AC voltage source in small circuit.



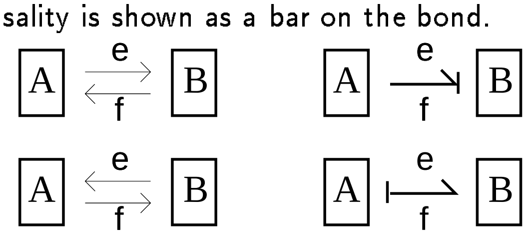
Causality
Purpose of Modelling is Insight

Causality is a fundamental concept in modelling. It shows dependencies between elements.
Causality is

- NOT a property of the bondgraph
- imposed by Sources
- easily analyzed by assignment procedure

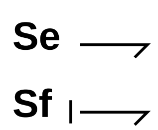
Causality
Element causality

Causality is shown as a bar on the bond.



See it as a nail, effort is imposed the flat end, flow at the point.

Sources have *mandatory* causality. Effort or flow given.

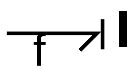


Resistances are indifferent to causality.

Causality
Integral causality

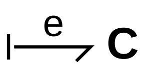
Storage elements are said to have *desirable* causality if it is integrating.
Numerical routines are designed to integrate, not differentiate.

- Effort causality for inertias



$$\Rightarrow f = \frac{1}{I}\int e\,dt$$

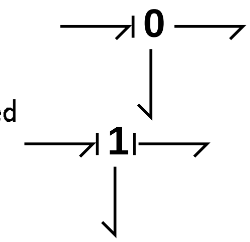
- Flow causality for capacitors



$$\Rightarrow e = \frac{1}{C}\int f\,dt$$

Causality
Causal constraints

Constitutive equations of junctions and transducers give constraints or propagation rules for causality.



- Common effort,

  one effort can be imposed
- Common flow,

  N-1 efforts imposed
- Transformer, same causality in and out

  $e_1 \rightarrow e_2$

  $f_2 \rightarrow f_1$

- Gyrator, switched causality

  $e_1 \rightarrow f_2$

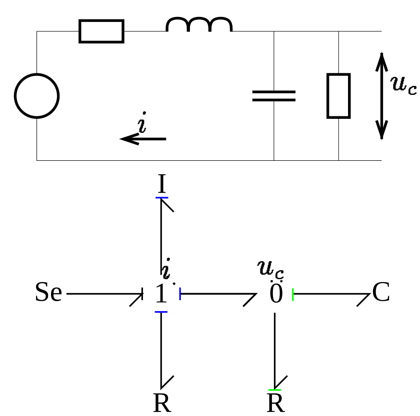
  $e_2 \rightarrow f_1$

Causality
Assigning procedure

1. Assign mandatory causality to sources.
2. Propagate causality through the junction structure, using the causal constraints on junctions, transformers and gyrators.
3. Assign desirable causality for one energy storage. Return to 2. and repeat until no more storage elements.
4. Remaining R-elements can be given any causality. Choose one and propagate as in 2. until all elements have causality.

Causality

Example: A simple electrical circuit.



1: Sources
(2): No propagation possible
3: Desirable causality, I + prop.
3: Choose another, C + prop.

Modeling Example
Issues in Modeling

The main issues when you want to make a model of a system are:

- Competence — Willems - MPUM
  Is the model adequate for our purpose?
- Simplicity — Occam's razor
  Is it the simplest model possible?

How can you check this?

- Causality analysis
  Are there any dependencies?
- Thought experiments
  Does the model behave like the real system?

Modelling example

Battery vacuum cleaner, electrical dynamics of motor neglected.

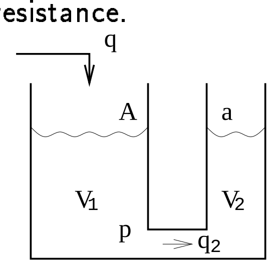
Analysis of models

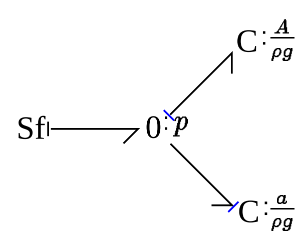
What information can you extract from a bond graph model?

- Causality analysis shows dependencies
- Index problems and Algebraic loops
- State equations are derived directly from a causal bond graph.
- Signal flow diagrams and
- Transfer functions can also be derived from a bond graph using Mason's rule. (Relative degree and Zero dynamics)
- Eigenvalue estimates, accurate to 10-20%
- Controllability/Observability? (Chapter 6.7, [4])

Analysis of models
Causality conflicts

Systems with conflicting/derivative causality seem to indicate high DAE index.
Example: Two connected tanks, no flow resistance.





$$\frac{dV_1}{dt} = q - q_2$$
$$\frac{dV_2}{dt} = q_2$$
$$p_1 = p_2 \Longrightarrow \frac{V_1}{A} = \frac{V_2}{a} \qquad (*)$$

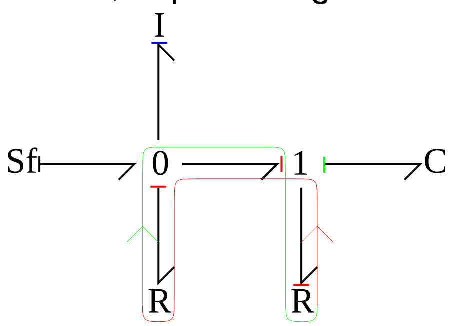
Equation (*) must be differentiated once to find q_2. This is an index 2 problem.

Analysis of models
Algebraic loops

Algebraic loops can be found by inspection of signal loops in the bondgraph. (Chap 6.5, [4])

Signal loop Closed causal path

Causal path Follows bonds with same causal orientation, stops at storage or source



Loop can be broken by inserting storage elements, cf. tearing.

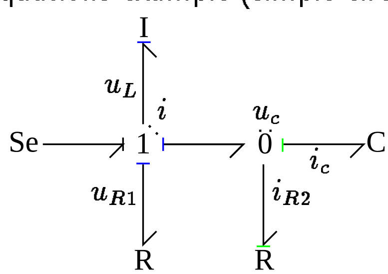
Analysis of models

Deriving system equations from a causal bondgraph.

1. Choose power state variables, outputs from independent energy storages.
2. Write the ODE for each state from the constitutive relation.
3. Follow the causality backwards to express the input in 2. as outputs from other elements.

Analysis of models

System equations example (simple circuit):



$$1: \mathrm{I}: i, \qquad \mathrm{C}: \; u_c$$

$$2: \frac{di}{dt} = u_L/L \qquad \frac{du_c}{dt} = i_c/C$$

$$3: u_L = V - u_{R1} - u_c =$$

$$= V - R_1 i - u_c$$

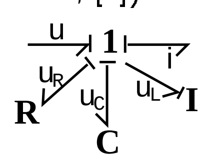
$$i_c = i - i_{R2} = i - u_c/R_2$$

$$\Rightarrow \frac{di}{dt} = \frac{1}{L}(V - R_1 i - u_c)$$

$$\frac{du_c}{dt} = \frac{1}{C}(i - \frac{u_c}{R_2})$$

Analysis of models

Eigenvalue estimation, assumption: parts of the bondgraph are loosely coupled.
Find subsystems that resemble resonance circuits. (Chapter 6.4, [4])

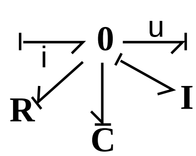


$$i = C\frac{du_C}{dt}, u_L = L\frac{di}{dt}, u_R = Ri,$$

$$u_C = u - u_L - u_R = U - LC\frac{d^2u_C}{dt^2} - RC\frac{du_C}{dt}$$

$$\Rightarrow (LCs^2 + RCs + 1)U_C(s) = U(s)$$

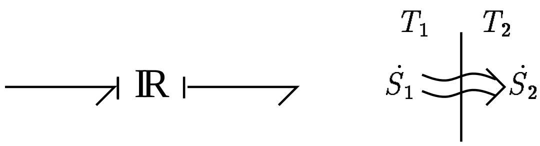
$$\omega_0^2 = \frac{1}{LC}, \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$



Advanced Bond Graphs
Multiports

Most energy transduction is associated with energy storage. This leads to general multiports, fields. Examples:

- Electrical transformer with flux energy, $\mathbb{I}$
- Ideal gas in closed container, $\mathbb{C}$
- Irreversible heat transfer, $\mathbb{R}$

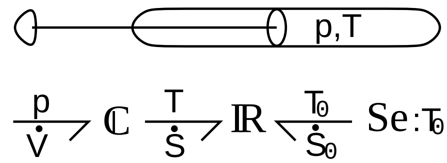


$$\dot{Q} = T_1\dot{S}_1 = T_2\dot{S}_2 \qquad \dot{S}_1 = h(T_1 - T_2)/T_1$$
$$\dot{Q} = h(T_1 - T_2) \qquad \dot{S}_2 = h(T_1 - T_2)/T_2$$
$$\Rightarrow \dot{S}_2 - \dot{S}_1 \geq 0$$

This is a nonlinear R-field. To certify second law, effort causality is *mandatory*.

Advanced Bond Graphs
Multiports

Example: A bicycle pump with blocked outlet.



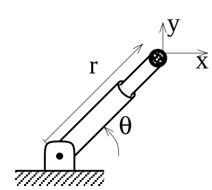
Advanced Bond Graphs
Multi-bonds and robotics

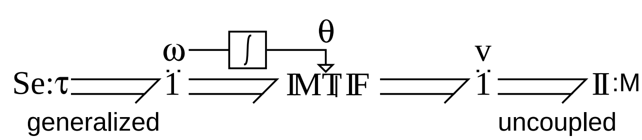
In two- and three-dimensional mechanics you need to have bonds with several efforts/flows, multibonds.
Motivation for bond graph robotics:

- 'Mechanics is hard for humans', Steve Crandle, MIT.
- Lagrangian becomes very complex algebraically, 6dof robot → 60000 terms
- Uncoupled dynamics are easy
- Handle in simple coordinate frame
  - Mechanics in uncoupled coord
  - Actuators in generalized coord

Advanced Bond Graphs
Multi-bonds and robotics

Example: (Ch 8.1 [4], Ch 23.2 [7])
Extendable robot arm.





Conclusions

- Bond graphs can be used to describe simple linear & nonlinear systems.
- \+ Displays causality & dependencies between parts of the system.
- \+ Systematic method, using physical analogies
- \+ Model subsystems independently
- \+ Simplifies nonlinear mechanics?
- – Tricky to learn, best suited for electromechanics.
- – Complex technical applications?
- – Not well suited for applications with multi-layered descriptions, simultaneous balance equations

1. References

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